JEE MAINS 2025 2024-25

Time: 60 Mins	**01 - Physics, 02 - Chemistry, 03 - Mathematics**	**Marks: 0**

Hints and Solutions

PHYSICS

01) Ans: **D)** The entire spectrum of visible light will come out of the water at various angles to the normal

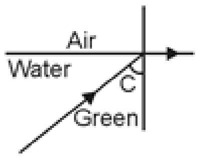
Sol: For total internal reflection of light take place, following conditions must be obeyed.

(i) The ray must travel from denser to rarer medium.

(ii) Angle of incidence (θ) must be greater than or equal to critical angle (C)

i.e. $C = \sin^{-1}\left[\frac{\mu_{rarer}}{\mu_{denser}}\right]$

Here, $\sin C = \frac{1}{n_{water}}$ and $n_{water} = a\frac{b}{\lambda^2}$

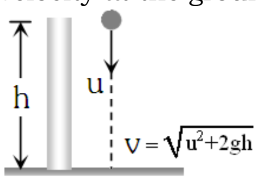


If frequency is less, λ is greater and thus, RI $n_{(water)}$ is less and therefore, critical angle increases.

Therefore, they do not suffer reflection and come out at angle less than 90^0.

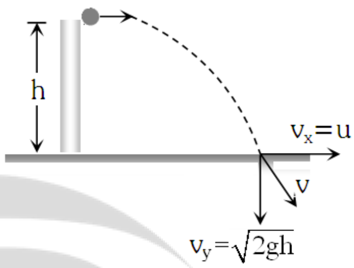
02) Ans: **A)** $1:1$

Sol: As soon as the particle is thrown in vertical downward direction with velocity u, then final velocity at the ground level



$v^2 = u^2 + 2gh \quad \therefore \quad v = \sqrt{u^2 + 2gh}$

and Another particle is thrown horizontally with same velocity, then at the surface of earth.



Horizontal component of velocity $v_x = u$

$\therefore$ Resultant velocity, $v = \sqrt{u^2 + 2gh}$

For both the particle final velocities, when they reach the earth's surface are equal.

03) Ans: **A)** less than the final pressure of B.

Sol: As A is compressed isothermally,

$\therefore P_1V = P_2\frac{V}{2} \Rightarrow P_2 = 2P_1$

and B is compressed adiabatically,

$\therefore P_1V^\gamma = P_2\left(\frac{V}{2}\right)^\gamma \Rightarrow P_2 = (2)^\gamma P_1$

As $\gamma > 1$, therefore $P_{2'} > P_2$ or $P_2 < P_2'$

04) Ans: **B)** A=208; Z=82

Sol: An $\alpha-$particle decay $\left({}^4_2He\right)$ reduces, mass number by 4 and atomic number by 2.

Decay of $6\alpha-$Particles results

${}^{232}_{90}Th \xrightarrow{6\alpha} {}^{232-24}_{90-12}Y = {}^{208}_{78}Y$

$A\beta-$decay does not produces any change in mass number but it increases atomic number by 1.

Decay of $4\beta-$particles results

${}^{208}_{78}Y \xrightarrow{4\beta} {}^{208}_{82}X$

Therefore, in the end nucleus $A = 208, Z = 82$

05) Ans: **B)** p

Sol: In isothermal process

pV=constant

$\Rightarrow pdV + Vdp = 0$ or $\left(\frac{dp}{dV}\right) = -\left(\frac{p}{V}\right)$

Bulk modulus, $B = -\left(\frac{dp}{dV/V}\right) = -\left(\frac{dp}{dV}\right)V$

$\Rightarrow B = -\left[\left(-\frac{p}{V}\right)V\right] = p$

$\therefore B = p$

Note Adiabatic bilk modulus is given by $B = \gamma P$.

06) Ans: **D)** 1.34 V/m

Sol: We know, $S_{av} = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{P}{4\pi R^2}$

$\Rightarrow E_0 = \sqrt{\frac{P}{2\pi R^2 \varepsilon_0 C}}$

$\Rightarrow E_0 = \sqrt{\frac{3}{2\times 3.14\times 100\times 8.85\times 10^{-12}\times 3\times 10^8}}$

$\Rightarrow E_0 = 1.34\ V/m$

07) Ans: **D)** both (2) and (3).

Sol: Consider the simple harmonic motions be represented by

$y_1 = a\sin\left(\omega t - \frac{\pi}{4}\right)$, $y_2 = a\sin\omega t$ and

$y_3 = a\sin\left(\omega t + \frac{\pi}{4}\right)$.

On superimposing, resultant S. H. M. will be

$y = a\left[\sin\left(\omega t - \frac{\pi}{4}\right) + \sin\omega t + \sin\left(\omega t + \frac{\pi}{4}\right)\right]$

$= a\left[2\sin\omega t\cos\frac{\pi}{4} + \sin\omega t\right]$

$= a[\sqrt{2}\sin\omega t + \sin\omega t] = a(1+\sqrt{2})\sin\omega t$

Resultant amplitude $= (1+\sqrt{2})a$

We know, energy is S. H. M. $\propto$ (Amplitude)^2

$\therefore \frac{E_{Resultant}}{E_{Single}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$

$\Rightarrow E_{Resultant} = (3+2\sqrt{2})E_{Single}$

08) Ans: **A)** $\frac{-GM}{R}$

Sol: $V_R = V_T - V_C$

$V_R =$ Potential due to remaining portion

$V_T =$ Potential due to total sphere

$V_C =$ Potential due to cavity

Radius of cavity is $\frac{R}{2}$.

Therefore, volume and mass is $\frac{M}{8}$.

$\therefore V_R = -\frac{GM}{R^3}\left[1.5R^2 - 0.5\left(\frac{R}{2}\right)^2\right] + \frac{G\left(\frac{M}{8}\right)}{\left(\frac{R}{2}\right)}\left(\frac{3}{2}\right) = -\frac{GM}{R}$

09) Ans: **C)** $\frac{aL^2}{2} + \frac{bL^3}{3}$

Sol: We know that change in potential energy of a system corresponding to a conservative internal force as $U_f - U_i = -W = -\int_i^f F.dr$

Given, $F = ax + bx^2$

We known that work done in stretching the rubber band by L is $|dW| = |Fdx|$

$\Rightarrow |W| = \int_0^L (ax+bx^2)dx = \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$

$= \left[\frac{aL^2}{2} - \frac{a\times(0)^2}{2}\right] + \left[\frac{b\times L^3}{3} - \frac{b\times(0)^3}{3}\right]$

$= |W| = \frac{aL^2}{2} + \frac{bL^3}{3}$

10) Ans: **A)** Straight line

Sol: The charged particle will be accelerated parallel (if it is a positive charge) or antiparallel (if it is a negative charge) to the electric field, i.e. the charged particle will move parallel or antiparallel to electric and magnetic field.

Hence, net magnetic force on it will be zero and its path will be a straight line

11) Ans: **B)** 748 J

Sol: Here, capacity of cylinder is

$= 67.2L$ and $\Delta T = 20^0 C$

At STP volume $= 22.4L$

Number of moles $= \frac{67.2}{22.4} = 3$

Now, change in heat is given as $\Delta Q = nC_V\Delta T$

By substituting the given values, we get

$\Delta Q = 3\times\frac{3R}{2}\times 20 \quad \left(\because \text{for He gas, } C_V = \frac{3}{2}R\right) = 90RJ$

Here, $R = 8.31\,J\,mol^{-1}K^{-1}$

$\therefore \Delta Q = 90\times 8.31 = 747.9J = 748J$

12) Ans: **B)** 5 : 8

Sol: Suppose, M_1 and M_2 be the magnetic moments of magnets and H the horizontal component of earth's field.

We have, $\tau = MH\sin\theta$. If ϕ is the twist of wire, then $\tau = C\phi$, C being restoring couple per unit twist of wire. $\Rightarrow C\phi = MH\sin\theta$

Here, $\phi_1 = (180^o - 30^o) = 150^o = 150\times\frac{\pi}{180}$ rad and

$\phi_2 = (270^o - 30^o) = 240^o_2 = 240\times\frac{\pi}{180}$ rad

Thus, $C\phi_1 = M_1 H\sin\theta$ (For deflection $\theta = 30^o$ of I magnet)

and $C\phi_2 = M_2 H\sin\theta$ (For deflection $\theta = 30^o$ of II magnet)

Dividing as,

$\frac{\phi_1}{\phi_2} = \frac{M_1}{M_2} \Rightarrow \frac{M_1}{M_2} = \frac{\phi_1}{\phi_2} = \frac{150\times\left(\frac{\pi}{180}\right)}{240\times\left(\frac{\pi}{180}\right)} = \frac{15}{24} = \frac{5}{8}$

$\Rightarrow M_1 : M_2 = 5 : 8$

13) Ans: **D)** 0.065 H
Sol: Resultant voltage,
$V^2 = V_R^2 + V_L^2 \Rightarrow 220^2 = 80^2 + V_L^2$
By solving, we get
$V_L \approx 205\,V \quad \Rightarrow X_L = \frac{V_L}{I} = \frac{205}{10} = 20.5\Omega = \omega L$

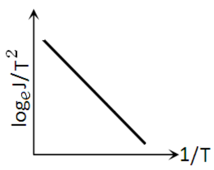
$\therefore L = \frac{20.5}{2\pi \times 50} = 0.065\,H$

14) Ans: **A)** 16 cm
Sol: Moment of inertia of hollow cylinder about its axis is $I_1 = \frac{M}{2}\left(R_1^2 + R_2^2\right)$
Where, R_1 = inner radius and R_2 = outer radius.
Moment of inertia of thin hollow cylinder of radius R about its axis is. $I_2 = MR^2$
Here, $I_1 = I_2$ and both cylinders have same mass (M).
So, we have $\frac{M}{2}\left(R_1^2 + R_2^2\right) = MR^2$
$\left(10^2 + 20^2\right)/2 = R^2 \quad \therefore R^2 = 250 = 15.8 \quad \Rightarrow R \approx 16\,cm$

15) Ans: **D)** 2.47 hr
Sol: Here, $mL = \frac{KA\Delta\theta\ t}{\Delta x}$
$\Rightarrow 500 \times 80 = \frac{0.0075 \times 75 \times (40 - 0)t}{5}$
$\Rightarrow$ t = 8.9×10^3 s = 2.47 hr.

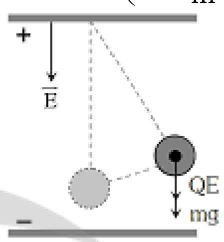
16) Ans: **A)** 0.21 V
Sol: Given that, L = 40 m,
v = 1080 km h^{-1} = 300 m s^{-1} and B = 1.75×10^{-5} T
$\Rightarrow e = Blv = 1.75 \times 10^{-5} \times 40 \times 300 = 0.21\,V$

17) Ans: **A)** 0.02
Sol: The relation between current (I) flowering through a conducting wire and drift velocity of electrons (v_d) is given as $I = neAv_d$
where, n is the electron density and A is the area of cross-section of wire.
$\Rightarrow v_d = \frac{I}{neA}$
By substituting the values, we get
$\Rightarrow v = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$
$\therefore v = \frac{1.5 \times 10^{-3}}{72} m/s = 0.2 \times 10^{-4} m/s$ or
$v = 0.02\,mm/s$



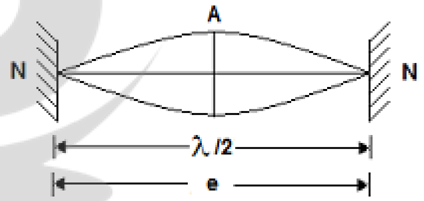
18) Ans: **D)**
Sol: By the Richardson-Dushman equation,
$J = AT^2 e^{-b/T}$
By taking log of this equation, $\log_e \frac{J}{T^2} = \log_e A - \frac{b}{T}$
It means graph between $\log_e \frac{J}{T^2}$ and $\frac{1}{T}$ will be a straight line having negative slope and positive intercept ($\log_e A$) on $\log_e \frac{J}{T^2}$ axis.

19) Ans: **B)** $\left(\frac{g}{g + \frac{qE}{m}}\right)^{1/2}$



Sol:
Here, net downward force mg' = mg + QE
$\Rightarrow$ Effect acceleration, $g' = \left(g + \frac{QE}{m}\right)$
$\therefore$ Time period, $T = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{\left(g + \frac{QE}{m}\right)}}$

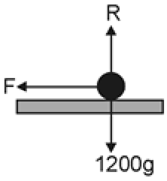
20) Ans: **D)** (i) 78.75 m/s (ii) 248 N



Sol:
In fundamental mode of vibration, $\frac{\lambda}{2} = 1o$ or $\lambda = 21$
$v = v\lambda \qquad \Rightarrow v = 45\,Hz \qquad \ldots($
As $\mu = \frac{m}{1} = 4 \times 10^{-2} kg/m \qquad \Rightarrow m = 3.5 \times 10^{-2}$
$\therefore 1 = \frac{m}{\mu} = \frac{3.5 \times 10^{-2}}{4 \times 10^{-2}} = 0.875\,m$
Now, For Eq. (i)
(i) $v = 45 \times 2 \times 0.875 = 78.75\,m/s$
$[\because \lambda = 21]$
(ii) As $v = \sqrt{\frac{T}{\mu}} \qquad \Rightarrow T = v^2(\mu) = (78.75)^2 \times 4.0$
$= 248.06\,N$

21) Ans: **D)** dimensionally correct only.
Sol: Equation $\tan\theta = \frac{rg}{v^2}$ is dimensionally correct as both sides are dimensionless but numerically wrong because the correct equation is $\tan\theta = \frac{v^2}{rg}$.

22) Ans: **7.84** Sol: The sketch shows the forces acting on the car when it is skidding.



As the vertical forces are in equilibrium $R = 1200g$

Since the car is skidding

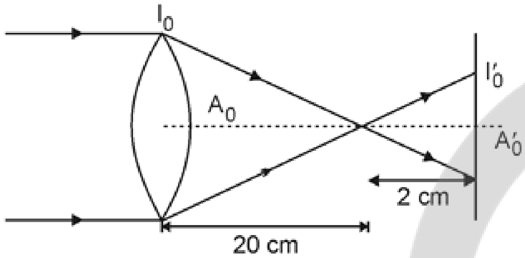
$F = \mu R = 0.8 \times 1200g = 9408N$

The friction F causes deceleration a which is given by Newton's Second Law, $F = ma = -9408 = 1200a$

Therefore, $a = -\frac{9408}{1200} = -7.84$

The deceleration is $7.84 ms^{-2}$

23) Ans: **130.0** Sol:



$\frac{A_0'}{A_0} = \left(\frac{2}{20}\right)^2 = \frac{1}{100} \Rightarrow A_0' = \frac{A_0}{100} \Rightarrow P = I_0 A_0 = I_0' A_0'$

$\Rightarrow I_0' = \frac{I_0 A_0}{\frac{A_0}{100}} \Rightarrow 100 I_0 = 130 kW/$

24) Ans: **3.13** Sol: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1$

$= P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$P_1 + \rho g h_1 = P_2 + \rho g h_2$

$v_2 = sqrt(2gh_1)$

$v_2 = sqrt\left(2(9.81 m/s^2)(0.5m)\right)$

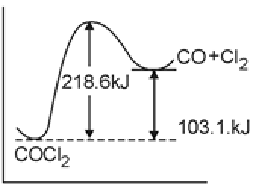
$v_2 = 3.13 m/s$

25) Ans: **3** Sol: Terminal velocity is given by

$v_T = \frac{2}{9}\frac{r^2}{\eta}(d-\rho)g \Rightarrow \frac{v_P}{v_Q} = \frac{r_P^2}{r_Q^2} \times \frac{\eta_Q}{\eta_P} \times \frac{(d-\rho_P)}{(d-\rho_Q)}$

$= \left(\frac{1}{0.5}\right) \times \left(\frac{2}{3}\right) \times \frac{(8-0.8)}{(8-1.6)} = 4 \times \frac{2}{3} \times \frac{7.2}{6.4} = 3$

CHEMISTRY



26) Ans: **A)**

Sol: By Arrhenius equation,

$\log_{10} k = -\frac{E_a}{2.303RT} + \log A$

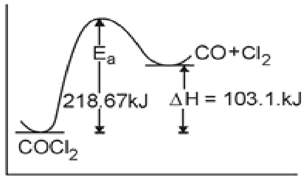
$\therefore \frac{E_a}{2.303} = 11420K \quad \Rightarrow E_a = 11420 \times 2.303$

$= 11420 \times 2.303 \times 8.314 \times 10^{-3} kJ\,mol\,K^{-1}K$

$= 218.67 kJ\,mol^{-1}$

Given reaction is endothermic, $\Delta H = +103.1 kJ$.

Therefore, reaction-coordinate diagram is



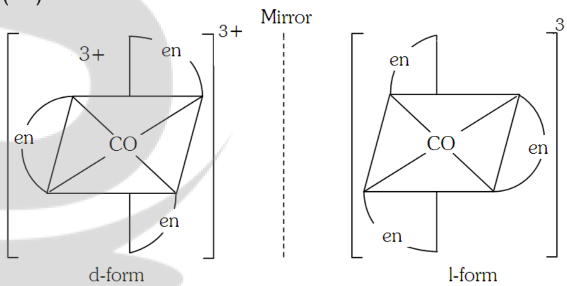
27) Ans: **C)** 9σ -bonds, 1π -bond and 2 lone pairs

Sol: $CH - C(:\ddot{O}H) = CH_2$

$9\sigma, 1\pi$ and 2Lp

28) Ans: **C)** tris-(ethylenediamine) cobalt (III) bromide

Sol: $[Co(en)_3]^{3+}$ i.e. tris-(ethylenediamine) cobalt (III) bromide



29) Ans: **B)** 58

Sol: 116 mg compounds means 116×10^{-3} gm compound as 1 mg contains 10^{-3} gm.

Mol. wt. of compound

$= \frac{\text{Mass of the substance}}{\text{Volume of the vapour at S.T.P.}} \times 22400$

$= \frac{116 \times 10^{-3}}{44.8} \times 22400 = 57.99\%$ or 58.0%

30) Ans: **C)** H_2O_2

Sol: Hydrogen peroxide (H_2O_2) acts as an oxidizing as well as reducing agent.

31) Ans: **B)** Ag and Au.

Sol: Au and Ag settle down below the anode because anode mudduring is the process of electrolytic refining of copper.

32) Ans: **B)** $Li_2^- < Li_2^+ < Li_2$

Sol:

$Li_2 (3+3=6) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2;$

$BO = \frac{N_b - N_a}{2} = \frac{4-2}{2} = 1$

$Li_2^- (3+3-1=5) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^1;$

$BO = \frac{3-2}{2} = 0.5$

$Li_2^- (3+3+1=7) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^1;$

$BO = \frac{4-3}{2} = 0.5$

Stability order is $Li_2 > Li_2^+ > Li_2^-$ (because Li_2^- has more number of electrons in antibonding orbitals which destabilizes the species).

33) Ans: **C)** Hg

Sol: $6Hg + O_3 \rightarrow \underset{\text{Mercurous oxide}}{3Hg_2O}$

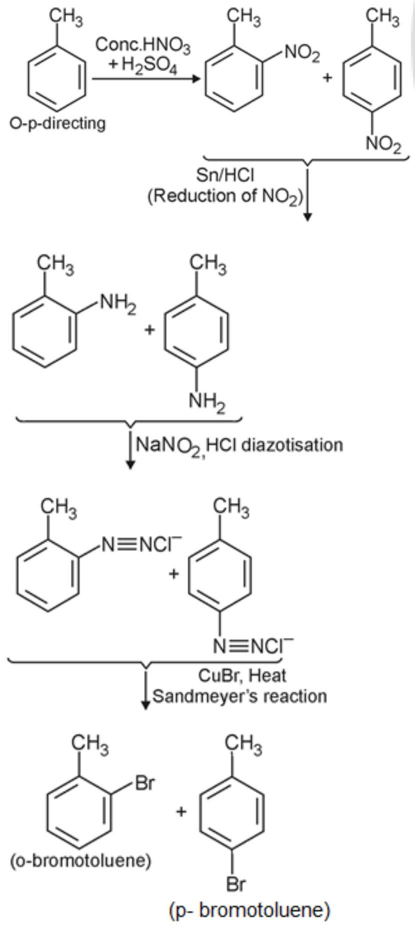
In this reaction, mercury loses its meniscus and starts sticking glass.

34) Ans: **D)** All of these.

Sol: Since all the ligands are weak, therefore they do to induce pairing of electrons so they show paramagnetism.

35) Ans: **A)** mixture of o- and p-bromotoluenes

Sol:



36) Ans: **A)** O - H

Sol: $CH_3CH_2OH \xrightarrow[\text{bond fission}]{\text{Heterolytic}} CH_3CH_2O^- + H^+$

37) Ans: **C)** aaaeee

Sol: The aaaeee form is the most powerful insecticide form of $C_6H_6Cl_6$.

38) Ans: **A)** 2.16 g

Sol: $\underset{2 \times 276\,gm}{2Ag_2CO_3} \xrightarrow{\Delta} \underset{4 \times 108\,gm}{4Ag} + 2CO_2 + O_2$

As $2 \times 276\,gm$ of Ag_2CO_3 gives $4 \times 108\,gm$,

$\therefore 1\,gm$ of Ag_2CO_3 gives $= \frac{4 \times 108}{2 \times 276}$.

$\therefore 2.76\,gm$ of Ag_2CO_3 gives

$= \frac{4 \times 108 \times 2.76}{2 \times 276} = 2.16\,gm$

39) Ans: **D)** optically active acid.

Sol: Glycine does not contain the chiral carbon, thus it is not optically active acid.

$$H_2N - \underset{H}{\overset{H}{C}} - COOH$$

(Glycine)

40) Ans: **D)** $-0.69^{o}C$

Sol: From given, $\Delta T_f = \frac{1000 \times 1.86 \times 17}{46 \times 1000} = 0.69^{o}C$

$T_f = 0 - 0.69 = -0.69^{o}C$

41) Ans: **B)** HNO_3

Sol: The main constituent of air nitrogen (78%) and oxygen (21%).

Only N_2 reacts with three moles of H_2 in the presence of a catalyst to give NH_3 (ammonia) which is a gas having basic nature.

On oxidation NH_3 gives NO_2 which is a part of acid rain.

Therefore the compounds A to D are as

$A = NH_4NO_2;\ \ B = N_2;\ \ C = NH_3;\ \ D = HNO_3$

42) Ans: **B)** 25:1

Sol: Energy change, $\Delta E = E_f - E_i$

$\Delta E = 2.18 \times 10^{-18} J \left(\frac{1}{n_i^2} - \frac{1}{n_f^2} \right)$

When $n_i = 5$ and $n_f = \infty$, Energy change,

$\Delta E = 2.18 \times 10^{-18} J \left(\frac{1}{5^2} - \frac{1}{\infty} \right) = 0.0872 \times 10^{-18} J$

When, $n_i = 1$ and $n_f = \infty$, energy change,

$\Delta E' = 2.18 \times 10^{-18} J \left(\frac{1}{1^2} - \frac{1}{\infty} \right) \Rightarrow \Delta E' = 2.18 \times 10^{-18} J$

$\frac{\Delta E'}{\Delta E} = \frac{2.18 \times 10^{-18}}{0.0872 \times 10^{-18}} = 25$

Hence, energy required to remove an electron from first orbit is 25 times than that required to remove an electron from fifth orbit.

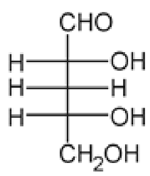
43) Ans: **A)** -372 kcal

Sol: Given, (i) $2C+3H_2 \rightarrow C_2H_6;\quad \Delta H=-21.1$
(ii) $C+O_2 \rightarrow CO_2;\quad \Delta H=-94.1$
(iii) $H_2+\frac{1}{2}O_2 \rightarrow H_2O;\ \Delta H=-68.3$
Eqa. 2 (ii) +3 (iii) - (i)
$C_2H_6+\frac{3}{2}O_2 \rightarrow 2CO_2+3H_2O$
$\Delta x=2(-=94.1)+3(-68.3)-(-21.1)\ =-3372$ kcal

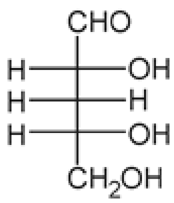
44) Ans: **A)** 11, 12, 18
Sol: For $X,(IE)_2$ is very high, thus X^+ attains inert gas configuration.
It is Na.
$Na(11) \rightarrow Na^+ + e^-$
$[Ne]3s^1 \quad [Ne]$
Stable, $(IE)_2$ is very high
For $Y,(IE)_3$ is very high thus, Y^{2+} attains inert gas configuration.
It is Mg.
$Mg(12) \rightarrow Mg^{2+}+2e^-$
$[Ne]3s^2 \quad [Ne]$
Stable, $(IE)_3$ is very high.
For Z, $(IE)_1$ is very high. It is [Ar].

45) Ans: **A)** $CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$
Sol: On dehydrogenation, secondary alcohol gives acetone.
$$CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3 \xrightarrow[300^0C]{Cu} CH_3-\underset{\displaystyle O}{\underset{||}{C}}-CH_3+H_2$$

46) Ans: **B)** $H_2C=CH-C\equiv CH$
Sol: $\overset{sp^2}{CH_2}=\overset{sp^2}{CH}-\overset{sp}{C}\equiv\overset{sp}{CH}$



47) Ans: **B)**
Sol: Formation of triacetate shows the presence of only three $-OH$ groups. Oxidation with Br_2-water shows the presence of a CHO group. Reduction with HI to n-pentane, shows the presence of a straight chain of 5 carbons. Formation of only 1 molecule of HCOOH suggests that its structures is



48) Ans: **D)** All of above

49) Ans: **79.8** Sol: We know that,
$\Delta G^o=-2.303\,RT\log K$
Also, given equilibrium is
$2H_2O \rightleftharpoons H_3O^+ + \bar{O}H$
$[H^+][OH^-]=10^{-14}$ or $K=10^{-14}$
$\Rightarrow \Delta G^o=-2.303\times 8.314JK^{-1}mol^{-1}\times 298K\times\log 10^{-14}$
$=79881.8\,J\,mol^{-1}=79.8kJ$

50) Ans: **3** Sol: Here, degree of ionisation
$(\alpha)=\frac{\wedge_m}{\wedge^\infty}$
Let $\wedge m(HY)=x \ \Rightarrow \wedge m(HX)=\frac{X}{10}$
$\Rightarrow \frac{\wedge m(HX)}{\wedge m(HY)}=\frac{1}{10}=\frac{\alpha(HX)}{\alpha(HY)} \quad [\because \wedge^\infty(HX)=\wedge^\infty(HY)]$
Also, $K_a=(HX)=(0.01)[\alpha(HX)]^2$ ....(i)
$K_a=(HY)=(0.10)[\alpha(HY)]^2$
$=0.10[10\alpha(HX)]^2=10[\alpha(HX)]^2$ ....(ii)
$\Rightarrow \frac{K_a(HX)}{K_a(HY)}=\frac{0.01}{10}=\frac{1}{1000}$
$\Rightarrow \log K_a(HX)-\log K_a(HY)=-3$
$\Rightarrow -\log K_a(HX)-[-\log K_a(HY)]=3$
$\Rightarrow pK_a(HX)-pK_a(HY)=3$

MATHEMATICS

51) Ans: **A)** 2
Sol: As we know, the system of linear equations
$a_1x+b_1y+c_1z=0$
$a_2x+b_2y+c_2z=0$
$a_3x+b_3y+c_3z=0$
has a non-trivial solution, if
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}=0$$
Now, if the given system of linear equations
$x+3y+7z=0$
$-x+4y+7z=0,$
and $(\sin 3\theta)x+(\cos 2\theta)y+2z=0$
has non-trivial solution, then
$$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos 2\theta & 2 \end{vmatrix}=0$$
$\Rightarrow 1(8-7\cos 2\theta)-3(-2-7\sin 3\theta)+7(-\cos 2\theta-4\sin 3\theta)=0$
$\Rightarrow 8-7\cos 2\theta+6+21\sin 3\theta-7\cos 2\theta-28\sin 3\theta=0$

$\Rightarrow -7\sin 3\theta - 14\cos 2\theta + 14 = 0$

$\Rightarrow -7\left(3\sin\theta - 4\sin^3\theta\right) - 14\left(1 - 2\sin^2\theta\right) + 14 = 0$

$\left[\because \sin 3A = 3\sin A - 4\sin^3 A \text{ and } \cos 2A = 1 - 2\sin^2 A\right]$

$\Rightarrow 28\sin^3\theta + 28\sin^2\theta - 21\sin\theta - 14 + 14 = 0$

$\Rightarrow 7\sin\theta\left[4\sin^2\theta + 4\sin\theta - 3\right] = 0$

$\Rightarrow \sin\theta\left[4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3\right] = 0$

$\Rightarrow \sin\theta\left[2\sin\theta\left(2\sin\theta + 3\right) - 1\left(2\sin\theta + 3\right)\right] = 0$

$\Rightarrow (\sin\theta)(2\sin\theta - 1)(2\sin\theta + 3) = 0$

Now, either $\sin\theta = 0 \text{ or } \frac{1}{2}$

$\left[\because \sin\theta \neq -\frac{3}{2} \text{ as } -1 \leq \sin\theta \leq 1\right]$

In given interval $(0,\pi)$,

$\Rightarrow \sin\theta = \frac{1}{2} \quad \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad [\because \sin\theta \neq 0, \theta \in (0,\pi)]$

So, 2 solutions in $(0,\pi)$

52) Ans: **D)** $\left(0, \frac{e-1}{2}\right)$

Sol: Whenever we have linear differential equation containing inequality, we should always check for increasing or decreasing, i.e. for

$\frac{dy}{dx} + Py < 0 \Rightarrow \frac{dy}{dx} + Py > 0$

Multiply by integrating factor, i.e. $e^{\int pdx}$ and convert into total differential equation.

Here, $f'(x) < 2f(x)$, multiplying by $e^{-\int 2dx}$

$f'(x).e^{-2x} - 2e^{-2x}f(x) < 0 \Rightarrow \frac{d}{dx}\left(f(x).e^{-2x}\right) < 0$

$\therefore \phi(x) = f(x)e^{-2x}$ is decreasing for $x \in \left[\frac{1}{2}, 1\right]$

Therefore, when $x > \frac{1}{2}$

$\phi(x) < \phi\left(\frac{1}{2}\right) \Rightarrow e^{-2x}f(x) < e^{-1}.f\left(\frac{1}{2}\right)$

$\Rightarrow f(x) < e^{2x-1}.1, \quad \text{given } f\left(\frac{1}{2}\right) = 1$

$\Rightarrow 0 < \int_{1/2}^{1} f(x)dx < \int_{1/2}^{1} e^{2x-1}dx$

$\Rightarrow 0 < \int_{1/2}^{1} f(x)dx < \left(\frac{e^{2x-1}}{2}\right)_{1/2}^{1}$

$\Rightarrow 0 < \int_{1/2}^{1} f(x)dx < \frac{e-1}{2}$

53) Ans: **B)** $\frac{1}{(a+b)^3}$

Sol: $\frac{\sin^4 A}{a} + \frac{\cos^4 A}{b} = \frac{1}{a+b}$

$\Rightarrow \frac{(1-\cos 2A)^2}{4a} + \frac{(1+\cos 2A)^2}{4b} = \frac{1}{a+b}$

$\Rightarrow b(a+b)(1 - 2\cos 2A + \cos^2 2A)$
$+ a(a+b)(1 + 2\cos 2A + \cos^2 2A) = 4ab$

$\Rightarrow \{b(a+b) + a(a+b)\}\cos^2 2A + 2(a+b)(a-b)\cos 2A$
$+ a(a+b) + b(a+b) - 4ab = 0$

$\Rightarrow (a+b)^2\cos^2 2A + 2(a+b)(a-b)\cos 2A + (a-b)^2 = 0$

$\Rightarrow \{(a+b)\cos 2A + (a-b)\}^2 = 0 \quad \Rightarrow \cos 2A = \frac{b-a}{b+a}$

$\therefore \frac{\sin^8 A}{a^3} + \frac{\cos^8 A}{b^3} = \frac{(1-\cos 2A)^4}{16a^3} + \frac{(1+\cos 2A)^4}{16b^3}$

$= \frac{1}{16a^3}\left[1 - \frac{b-a}{b+a}\right]^4 + \frac{1}{16b^3}\left[1 + \frac{b-a}{b+a}\right]^4$

$= \frac{16a^4}{16a^3(b+a)^4} + \frac{16b^4}{16b^3(b+a)^4}$

$= \frac{1}{(b+a)^4}(a+b) = \frac{1}{(a+b)^3}$

54) Ans: **A)** $(-8, \infty)$

Sol: Given,

$(x+3)/(x-2) > 1/2$

$\Rightarrow 2(x+3) > x-2$

$\Rightarrow 2x + 6 > x - 2$

$\Rightarrow 2x - x > -2 - 6$

$\Rightarrow x > -8$

$\Rightarrow x \in (-8, \infty)$

55) Ans: **A)** $C > \frac{1}{2}$

Sol: The normal to the parabola $y^2 = 4ax$ in slope form is $y = mx - 2am - am^3$.

For the given curve $y^2 = x$, we will have

$4a = 1 \Rightarrow a = \frac{1}{4}$

$\therefore$ The equation of the normal is

$y^2 = mx - \frac{1}{2}m - \frac{1}{4}m^3$

If it passes through $(C, 0)$, then

$0 = mC - \frac{1}{2}m - \frac{1}{4}m^3 \Rightarrow m = 0$

$\Rightarrow C - \frac{1}{2} - \frac{1}{4}m^2 = 0 \Rightarrow m = \pm 2\sqrt{C - \frac{1}{2}}$

For three normals, value of m should be real,

$\therefore C > \frac{1}{2}$.

56) Ans: **A)** ${}^{30}C_{10}$

Sol: $(1-x)^{30} = {}^{30}C_0x^0 - {}^{30}C_1x^1 + {}^{30}C_2x^2$

$+......+(-1)^{30}\ {}^{30}C_{30}x^{30}$ ....(i)

$(x+1)^{30} = {}^{30}C_0x^{30} + {}^{30}C_1x^{29} + {}^{30}C_2x^{28}$

$+....+{}^{30}C_{10}x^{20} + ... + {}^{30}C_{30}x^0$ ....(ii)

By multiplying (i) and (ii) and equating coefficient of x^{20} on both sides,
we get required sum = coefficient of x^{20} in $(1-x^2)^{30} = {}^{30}C_{10}$

57) Ans: **B)** $-\frac{1}{10}$

Sol: Equation of radical axis is $S_1 - S_2 = 0$.

Here, $S_1 \equiv x^2 + y^2 - 3x - 4y + 5 = 0$

$S_2 \equiv x^2 + y^2 - \frac{7}{3}x + \frac{8y}{3} + \frac{11}{3} = 0$

$\therefore$ Radical axis is $-2x - 20y - 4 = 0$

$\therefore$ Gradient of radical axis $= -\frac{1}{10}$

58) Ans: **A)** $\frac{4}{(2n+1)\pi}$

Sol: $\tan(\cot x) = \cot(\tan x)$

$\Rightarrow \tan(\cot x) = \tan\left(\frac{\pi}{2} - \tan x\right)$

$\Rightarrow \cot x = n\pi + \frac{\pi}{2} - \tan x \Rightarrow \cot x + \tan x = n\pi + \frac{\pi}{2}$

$\Rightarrow \frac{2}{\sin 2x} = n\pi + \frac{\pi}{2}$ i.e. $\sin 2x = \frac{2}{n\pi + \frac{\pi}{2}} = \frac{4}{(2n+1)\pi}$

59) Ans: **A)** $\alpha = \beta = \cos\theta, \quad \gamma^2 = -\cos 2\theta$

Sol: $c = \alpha a + \beta b + \gamma(a \times b) \quad \Rightarrow c.a = \alpha$ and $c.b = \beta$

$\Rightarrow \alpha = \beta = \cos\theta$

Also, $1 = c.c$,

$\therefore [\alpha a + \beta b + \gamma(a \times b)].[(\alpha a + \beta b) + \gamma(a \times b)] = 1$

$\Rightarrow 2\alpha^2 + \gamma^2 (a \times b)^2 = 1 \quad \{\because \alpha = \beta\}$

$\Rightarrow 2\alpha^2 + \gamma^2\left[a^2b^2 - (a.b)^2\right] = 1 \quad \Rightarrow 2\alpha^2 + \gamma^2 = 1$

$\therefore \gamma^2 = 1 - 2\alpha^2 = 1 - 2\cos^2\theta = -\cos 2\theta$

60) Ans: **D)** $\frac{\sqrt{3}}{10}$

Sol: Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right)$

Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$

$\therefore u = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1} x$

$\therefore \frac{du}{dx} = \frac{1}{2} \times \frac{1}{(1+x^2)}$

Let $v = \tan^{-1}\left(\frac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Put $x = \sin\phi \Rightarrow \phi = \sin^{-1} x$

$v = \tan^{-1}\left(\frac{2\sin\phi\cos\phi}{\cos 2\phi}\right) = \tan^{-1}(\tan 2\phi)$

$= 2\phi = 2\sin^{-1} x$

$\frac{dv}{dx} = 2\frac{1}{\sqrt{1-x^2}}$

$\frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{\sqrt{1-x^2}}{4(1+x^2)}$

$\therefore \left(\frac{du}{dv}\right)_{\left(x=\frac{1}{2}\right)} = \frac{\sqrt{3}}{10}$

61) Ans: **D)** 16

Sol: $\because$ Mean$(\bar{x}) = \frac{6+10+7+13+a+12+b+12}{8} = 9$

$\Rightarrow 60 + a + b + 72$

$\Rightarrow a + b + 12$

$\sum x_i^2 = 6^2 + 10^2 + 7^2 + 13^2 + a^2 + b^2 + 12^2 + 12^2$

$= a^2 + b^2 + 642$

variance $(\sigma^2) = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \frac{37}{4}$

$\Rightarrow \frac{a^2 + b^2 + 642}{8} - (9)^2 = \frac{37}{4}$

$\Rightarrow \frac{a^2 + b^2 + 642}{8} = \frac{361}{4}$

$\Rightarrow a^2 + b^2 = 80$

$(a+b)^2 = a^2 + b^2 + 2ab \Rightarrow 144 = 80 + 2ab \Rightarrow 2ab = 64$

Now, $(a-b)^2 = a^2 + b^2 - 2ab = 80 - 64 = 16$

62) Ans: **C)** $\frac{4}{9}$ and $\frac{3}{2}$

Sol: Let X denote the number of heads tossed. So, X can take the values 0,1,2,3.
When a coin is tossed three times, we get Sample space,

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$\Rightarrow P(X=1) = P(\text{one head}) = P(HTT, THT, TTH) = \frac{3}{8}$

$P(X=2) = P(\text{two heads}) = P(HHT, HTH, THH) = \frac{3}{8}$

$\Rightarrow P(X=3) = P(\text{three heads}) = P(HHH) = \frac{1}{8}$

Thus, the probability distribution of X is

X	0	1	2	3
P(x)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Variance of $X = \sigma^2 = \sum X_i^2 p_i - \mu^2$ ...(i)

where, μ is the mean of X given by

$\mu = \sum X_i P_i = 0.\frac{1}{8} + 1.\frac{3}{8} + 2.\frac{3}{8} + 3.\frac{1}{8} = \frac{3}{2}$ ...(ii)

Now, $\sum X_I^2 P_i = 0^2.\frac{1}{8} + 1^2.\frac{3}{8} + 2^2.\frac{3}{8} + 3^2.\frac{1}{8} = 3$ ...(iii)

From Eqs. (i), and (iii), we get $\sigma^2 = \left(3 - \frac{3}{2}\right)^2 = \frac{9}{4}$

Therefore, standard deviation $= \sqrt{\sigma} = \sqrt{\frac{9}{4}} = \frac{3}{2}$

63) Ans: **A)** -13.5
Sol: Given that $x, 2x+2, 3x+3$ are in G.P.

$\therefore (2x+2)^2 = x(3x+3) \Rightarrow x^2 + 5x + 4 = 0$

$\Rightarrow (x+4)(x+1) = 0 \Rightarrow x = -1, -4$

Now, first term $a = x$

Second term $ar = 2(x+1) \Rightarrow r = \frac{2(x+1)}{x}$

then 4^{th} term $= ar^3 = x\left[\frac{2(x+1)}{x}\right]^3 = \frac{8}{x^2}(x+1)^3$

Now, putting $x = -4$

$\Rightarrow T_4 = \frac{8}{16}(-3)^3 = -\frac{27}{2} = -13.5$

64) Ans: **B)** $\frac{1}{2}$

Sol: $\int_0^\infty e^{-2x}(\sin 2x + \cos 2x)dx$

$= \left[-e^{-x}\frac{\cos 2x}{2}\right]_0^\infty - \int_0^\infty \left(-2e^{-2x}\right)\left(\frac{-\cos 2x}{2}\right)dx$

$+ \int_0^\infty e^{-2x}\cos 2x\, dx$

$\Rightarrow \int_0^\infty e^{-2x}(\sin 2x + \cos 2x)dx = \frac{1}{2}$

65) Ans: **D)** $\left(\frac{x^2}{2} - \frac{1}{4}\right)\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + c$

Sol: Let, I =

$\int x \sin^{-1} x dx = \frac{x^2}{2}\sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}}.\frac{x^2}{2}dx + c$

$I = \frac{x^2}{2}\sin^{-1} x - \frac{1}{2}\int -\frac{(1-x^2)+1}{\sqrt{1-x^2}}dx + c$

$I = \frac{x^2}{2}\sin^{-1} x + \frac{1}{2}\int \sqrt{1-x^2}\,dx - \frac{1}{2}\int \frac{1}{\sqrt{1-x^2}}dx + c$

$I = \frac{x^2}{2}\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + \frac{1}{4}\sin^{-1} x - \frac{1}{2}\sin^{-1} x + c$

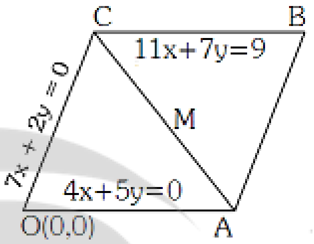
$I = \frac{x^2}{2}\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1} x$

$I = \left(\frac{x^2}{2} - \frac{1}{4}\right)\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2} + c$

66) Ans: **A)** 20
Sol: As 2 persons can drive the car, thus we have to select 1 from these two. This can be done in ${}^2C_1$ ways. Now, from the remaining 5 persons we have to select 2 which can be done in ${}^5C_2$ ways.
$\therefore$ Required number of ways in which the car can be filled is ${}^5C_2 \times {}^2C_1 = 20$.

67) Ans: **A)** $x - y = 0$
Sol: Equation of diagonal $11x + 7y = 9$ does not pass through origin, So it cannot be equation of the diagonal OB.
By solving the equation AC with the equations OA and OC,

$A\left(\frac{5}{3}, -\frac{4}{3}\right)$ and $C\left(\frac{-2}{3}, \frac{7}{3}\right)$



The middle point M is $\left(\frac{1}{2}, \frac{1}{2}\right)$

$\therefore$ The equation of OB is $y = x$ i.e. $x - y = 0$

68) Ans: **C)** $\frac{\pi}{4}$

Sol: Let

$S_\infty = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + ...$

$\therefore T_n = \cot^{-1} 2n^2$

$= \tan^{-1}\frac{1}{2n^2}$

$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left[\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right]$

$= \tan^{-1}(2n+1) - \tan^{-1}(2n-1)$

$\therefore S_n = \sum_{n=1}^{\infty}\left\{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\right\}$

$= \tan^{-1}\infty - \tan^{-1} 1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

69) Ans: **15** Sol:
$n(A-B) = n(A) - n(A \cap B) = 25 - 10 = 15$

70) Ans: **0.25** Sol: We have
$\alpha + \beta = 3/8 \text{ and } \alpha\beta = 27/8$

$\left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3} = \frac{\alpha+\beta}{(\alpha\beta)^{1/3}} = \frac{3/8}{(27/8)^{1/3}}$

$= \frac{3/8}{3/2} = \frac{3}{8} \times \frac{2}{3} = \frac{1}{4} = 0.25$

71) Ans: **180** Sol: Given a 5 digit number product is 36.
Multiple of $36 = 2^6 \times 3^2$

$= 2 \times 2 \times 3 \times 3$

We need to make five digit number by using the digits 2, 2, 3 & 3 and the fifth digit could be 7.

Case-I: 2, 2, 3, 3, 1

Total number of 5 digits numbers $\frac{5!}{2!2!} = 30$

Case-II: 4, 3, 3, 1, 1

Total number of 5 digits numbers $= \frac{5!}{2!2!} = 30$

Case-III: 6, 2, 3, 1, 1

Total number of 5 digits numbers $= \frac{5!}{2!} = 60$

Case-IV: 9, 2, 2, 1, 1

Total number of 5 digits numbers $= \frac{5!}{2!2!} = 30$

Case-V: 4, 9, 1, 1, 1

Total number of 5 digits numbers $= \frac{5!}{3!} = 20$

Case-VI: 6, 6, 1, 1, 1

Total number of 5 digits numbers $= \frac{5!}{3!2!} = 10$

Total number of 5 digits numbers = 30+30+60+30+20+10=180.

72) Ans: **1** Sol: Here, $\begin{vmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{vmatrix} = 0$

$\Rightarrow \alpha^4 - 2\alpha^2 + 1 = 0 \quad \Rightarrow \alpha^2 = 1 \quad \Rightarrow \alpha = \pm 1$ but $\alpha = 1$ not possible

$\therefore \alpha = -1$ So, $1 + \alpha + \alpha^2 = 1$

73) Ans: **1** Sol:

$\left[\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right]^{3/4} = [\cos\pi + i\sin\pi]^{1/4}$

Since the expression has only 4 different roots, thus on putting $n = 0, 1, 2, 3$ in

$\cos\left[\frac{2n\pi + \pi}{4}\right] + i\sin\left[\frac{2n\pi + \pi}{4}\right]$ and multiplying them, we get,

$= \left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]\left[\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right]$

$\left[\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right]\left[\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right]$

$= \left[\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]\left[-\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right]\left[-\frac{1}{\sqrt{2}} + i\frac{-1}{\sqrt{2}}\right]\left[\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right]$

$= \left(-\frac{1}{2} - \frac{1}{2}\right)\left(-\frac{1}{2} - \frac{1}{2}\right) = (-1)(-1) = 1$

74) Ans: **6** Sol:

Coordinates of $C = \left(\frac{8}{5}, \frac{14}{5}\right)$

Line 2x+y=k passes through c

$\Rightarrow \frac{2 \times 8}{5} + \frac{14}{5} = k \Rightarrow k = 6$

75) Ans: **6** Sol: Let

$p(E_1) = x, p(E_2) = y$ and $p(E_3) = z$

$\alpha = p\left(E_1 \cap \bar{E}_2 \cap \bar{E}_3\right) = p(E_1).p\left(\bar{E}_2\right).p\left(\bar{E}_3\right)$

$\Rightarrow \alpha = x(1-y)(1-z)$ .... (i)

Similiarly,

$\beta = (1-x).y(1-z)$ .... (ii)

$\gamma = (1-x)(1-y)z$ ... (iii)

$p(1-x)(1-y)(1-z)$ .... (iv)

From (i) and (iv),

$\frac{x}{1-x} = \frac{\alpha}{p} \Rightarrow x = \frac{\alpha}{\alpha + p}$

From (iii) and (iv),

$\frac{z}{1-z} = \frac{\gamma}{p} \Rightarrow z = \frac{\gamma}{\gamma + p}$

$\frac{p(E_1)}{p(E_2)} = \frac{x}{z} = \frac{\frac{\alpha}{\alpha+p}}{\frac{\gamma}{\gamma+p}} = \frac{\frac{\gamma+p}{\gamma}}{\frac{\alpha+p}{\alpha}} = \frac{1+\frac{p}{\gamma}}{1+\frac{p}{\alpha}}$ .... (v)

Given that

$(\alpha - 2\beta)p = \alpha\beta \Rightarrow \alpha p = (\alpha + 2p)\beta$ .... (vi)

$(\beta - 3\gamma)p = 2\beta\gamma \Rightarrow 3\gamma p = (p - 2\gamma)\beta$ ... (vii)

From (vi) and (vii)

$\frac{\alpha}{3\gamma} = \frac{\alpha + 2p}{p - 2\gamma} \Rightarrow p\alpha - 6p\gamma = 5\gamma\alpha$

$\alpha\frac{p}{\gamma} - \frac{6p}{\alpha} = 5 \Rightarrow \frac{p}{\gamma} + 1 = 6\left(\frac{p}{\alpha} + 1\right)$ .... (viii)

From (v) and (viii),

$\frac{p(E_1)}{p(E_3)} = 6$